IDENTIFICATION OF CONTINUOUS-TIME SYSTEMS

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We must

know

an object (system)

before we venture to

handle it.

Modeling and Identification are tools in our efforts to develop knowledge about an object (system) :

two *very vast and well established* fields

Modeling (physical systems)

- Natural laws
- Generic mathematical description

Identification

- *Determination of the values of parameters in the generic model using measurements*

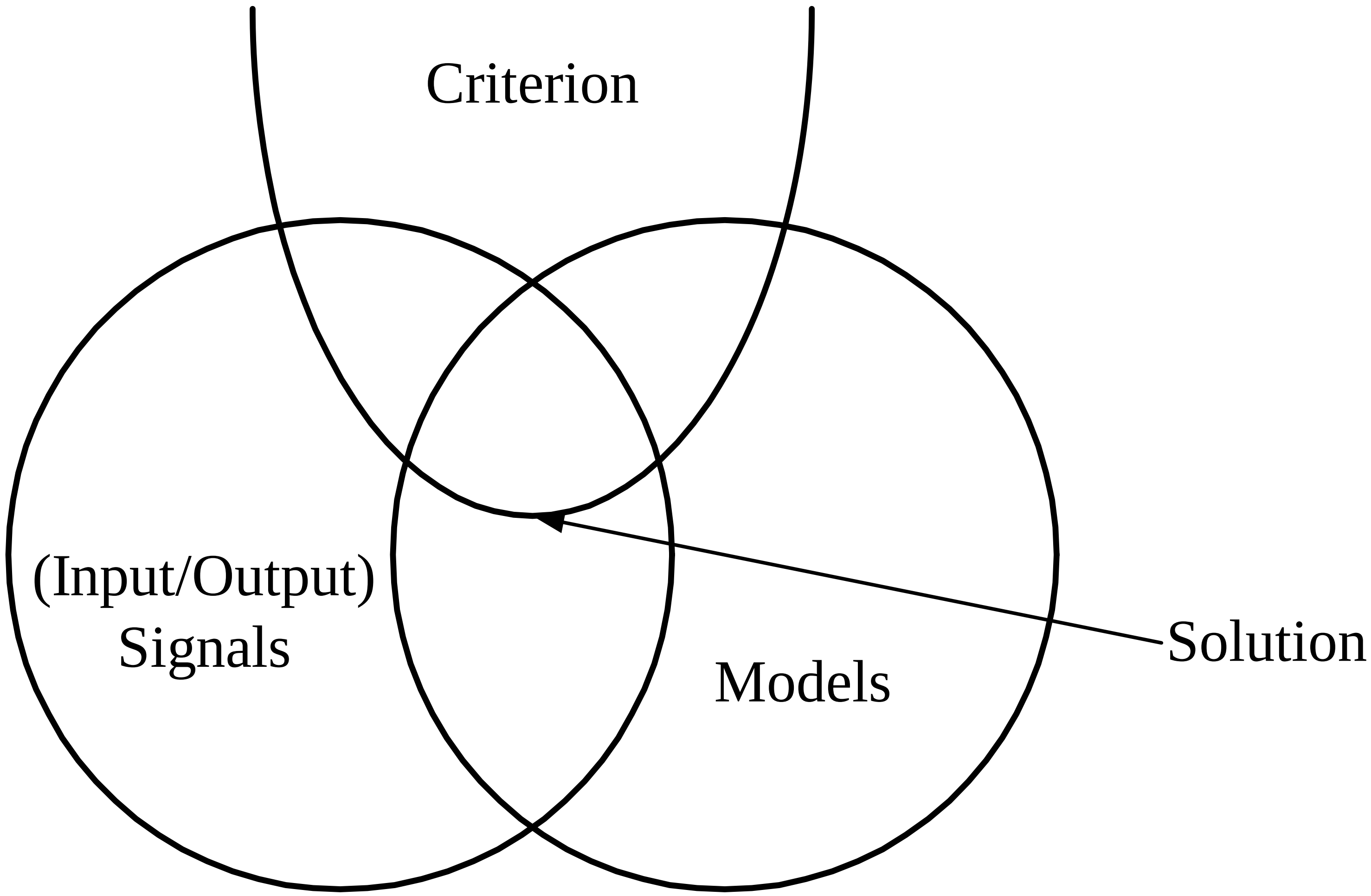


Fig. 1. The system identification problem.

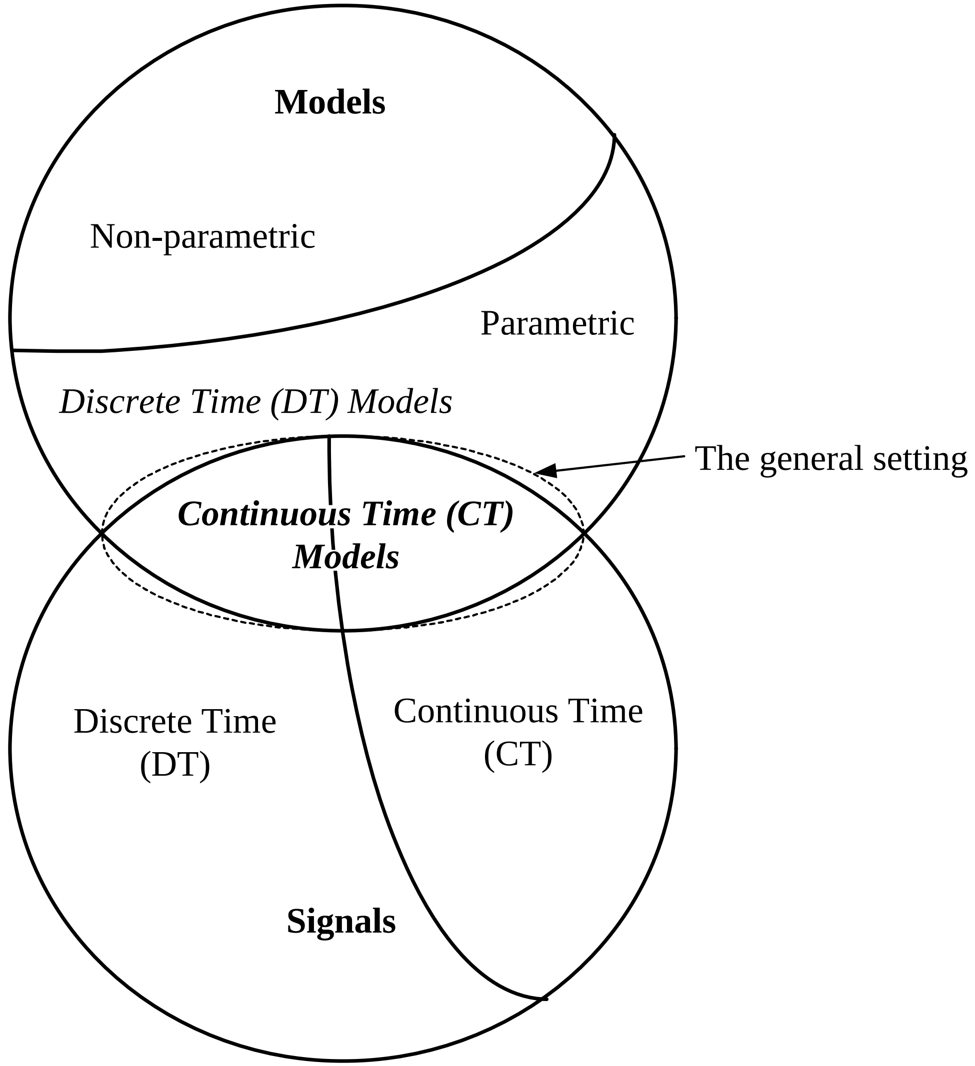


Fig. 2. The general setting for identification of Continuous Time Systems.

Two Approaches

- *Indirect approach* via Discrete-Time (DT) models

First identify a DT model and convert to CT

- *Direct approach* through Continuous-Time (CT) models

Identify the CT model directly

Why CT models?

- Laws of physical systems are in CT
- Therefore, CT models are native to CT domain.
- CT models provide good insight into the system properties
- Continuous-time models are useful in applications such as fault diagnosis

CT models preserve available knowledge

$$G(s) = K/[(s+a)(s+b)(s+c)]$$

$$G_z(z) = (b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3})/(1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3})$$

DT approach ignores available knowledge and assumes full 'ignorance'

Discretization may render CT models non-minimum phase

$$G(s) = \frac{a}{s(s+a)}$$

zero order hold

$$G_z(z) = \frac{Az + B}{C(z-1)(z-D)}$$

$$A = e^{-aT} + aT - 1$$

$$B = 1 - e^{-aT} - aT\, e^{-aT}$$

$$C = a \qquad D = e^{-aT}$$

The zero of $G_z(z)$

$$z_{z1} = -(B/A)$$

$$a = 1$$

lies within the unit circle in the *z*-plane for $T_S > 2$

moves far outside the unit circle for practically interesting values of $T_S < 0{,}5$

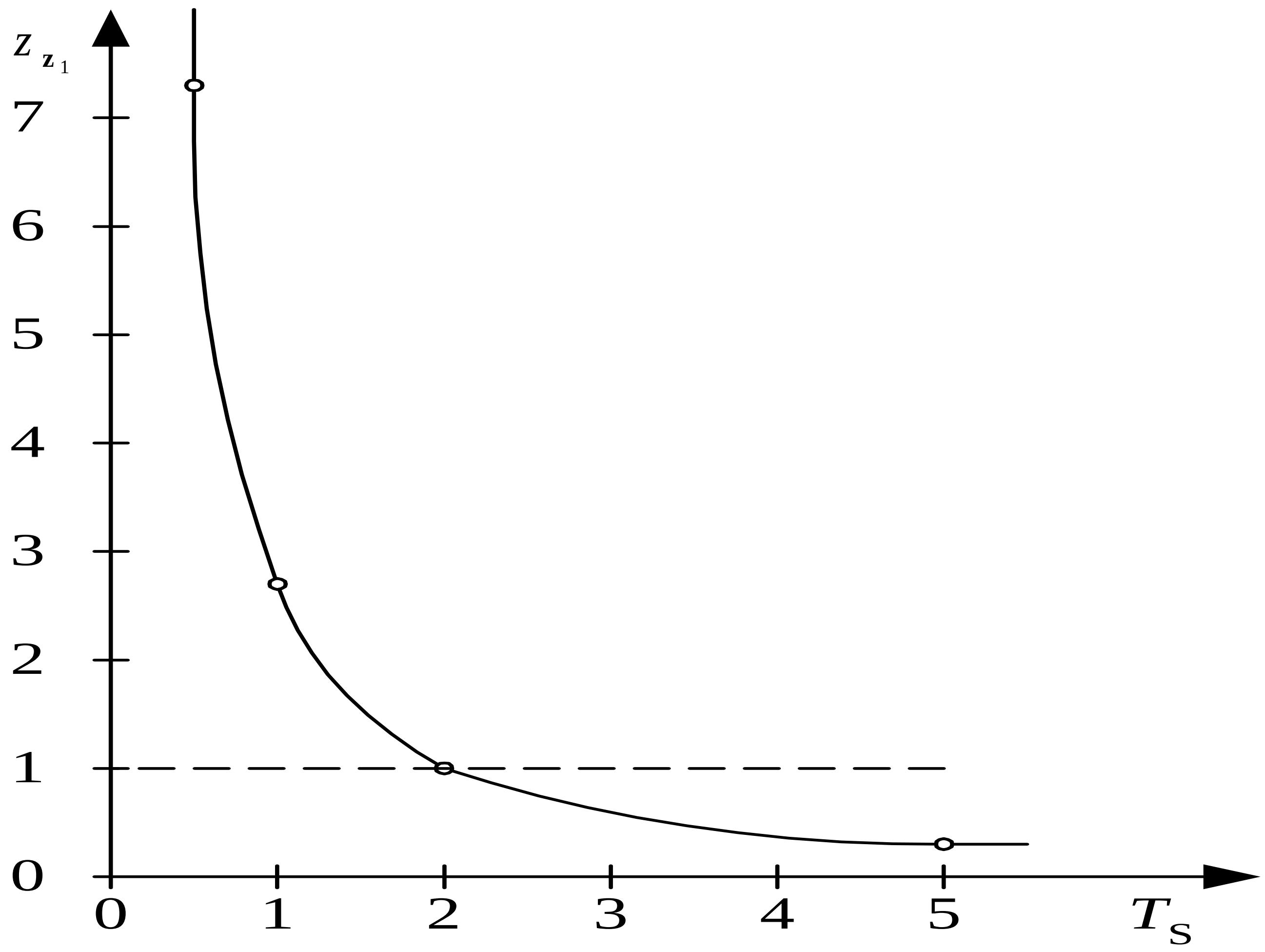


Fig. 3. Dependence of the zero z_{z1} of $G_z(z)$ on the sampling time.

- Discretization gives rise to undesirable sensitivity problems at high sampling rates
- Increasing sampling rate in a DT model does not lead to the original CT model
- Discretization of a CT model gives a unique DT model, but the reverse action does not lead to its native CT model

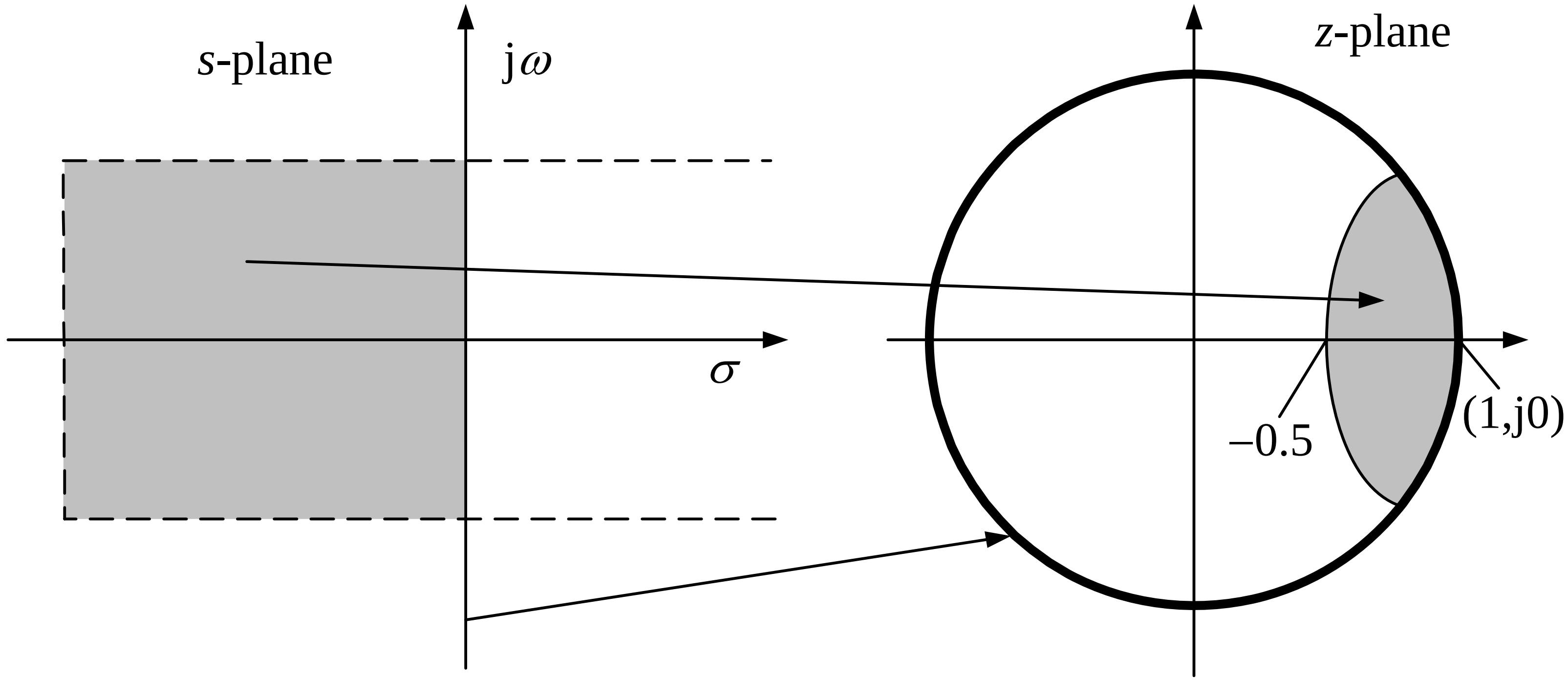


Fig. 4. The region of normal operation in the *z*-plane.

A black hole in the z-plane

- **With increasing sampling frequency the point (1,*j*0) in the *z*-plane tends to become a black hole!**

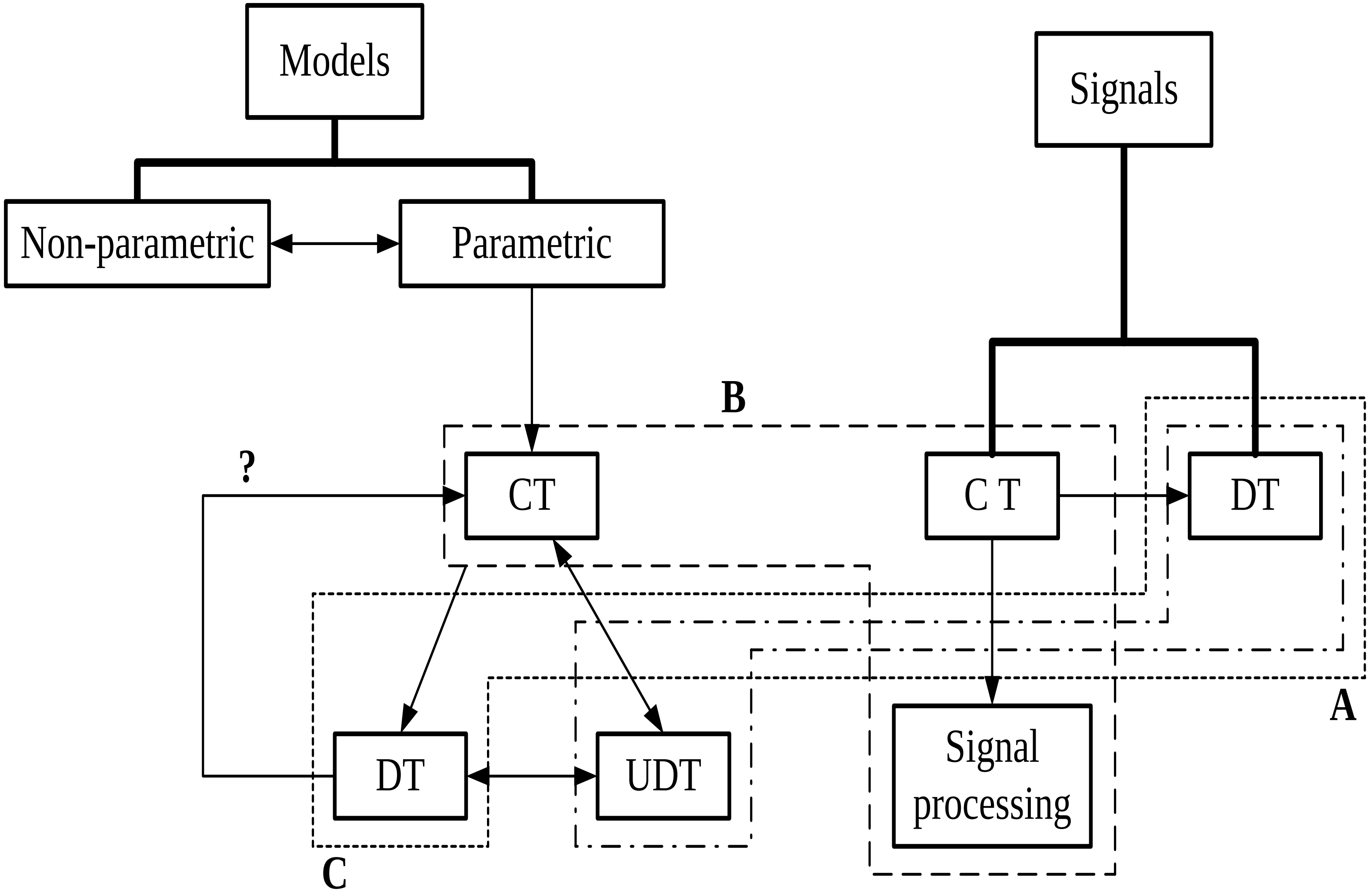


Fig. 5. Approaches to Identification of Continuous Time Systems in the general setting.

Major problem with CT models: Derivative terms

$$Y(s)/U(s) = G(s) = b/(1+as)$$

$$a\mathrm{d}y(t)/\mathrm{d}t + y(t) = bu(t)$$

sample at $t_k, k = 1, 2, 3, \ldots$

$$av(k) + y(k) = bu(k)$$

[*Transposed vector of measurements*]
[*Parameter vector*]
=[*A single measurement of the output*]
Or

$$\mathbf{m}_k{}^{\mathrm{T}}\boldsymbol{\theta} = y_k$$

[*Matrix of measurements*]
[*Parameter vector*]
=[*Output measurement vector*]
Or

$$\mathbf{M}\boldsymbol{\theta} = \mathbf{y}$$

Major approaches

$$a\mathrm{d}y(t)/\mathrm{d}t + y(t) = bu(t)$$

Modulating functions

$$\{\varphi_n(t)\}, n=1,2,\ldots, t\in[0,t_0], \varphi_n(0)=\varphi_n(t_o)=\mathrm{d}\varphi_n/\mathrm{d}t\Big|_0=\mathrm{d}\varphi_n/\mathrm{d}t\Big|_{t_0}=0, \quad n=1,2,\ldots.$$

$$a\int_0^{t_0}\varphi_n(t)\frac{\mathrm{d}y}{\mathrm{d}t}\mathrm{d}t+\int_0^{t_0}\varphi_n(t)y(t)\mathrm{d}t=b\int_0^{t_0}\varphi_n(t)u(t)\,\mathrm{d}t$$

$$\int_0^{t_0}\varphi_n(t)y(t)\,\mathrm{d}t-a\int_0^{t_0}\frac{\mathrm{d}\varphi_n}{\mathrm{d}t}y(t)\,\mathrm{d}t=b\int_0^{t_0}\varphi_n(t)u(t)\,\mathrm{d}t,\ n=1,\ 2,\ \ldots.$$

$$[\quad \int_0^{t_0} \frac{\mathrm{d}\varphi_n}{\mathrm{d}t} y(t)\,\mathrm{d}t \qquad \int_0^{t_0} \varphi_n(t)\, y(t)\,\mathrm{d}t],\ \ n = 1,\ 2,\ \ldots.$$

$$[a\ b]^{\mathrm{T}}$$

$$\int_0^{t_0} \varphi_n(t)\, u(t)\,\mathrm{d}t \qquad\qquad n = 1,\ 2,\ \ldots.$$

The Poisson moment functionals (PMF)

Modulating function:

Inverse Laplace transform of $1/(s+\lambda)^{i+1}$.

$$M_i\{\mathrm{d}y/\mathrm{d}t\} \triangleq \int_0^t [(t-\tau)^i / i!] \exp[-\lambda(t-\tau)] \frac{\mathrm{d}y}{\mathrm{d}\tau} \mathrm{d}\tau$$

$$[M_i\{y(t)\} - \lambda_i M\{y(t)\} - p_i(t) \;\; y(0)\, M_i\{u(t)\}]$$

Outputs at the various stages of a filter chain
with identical stages,
each with transfer function $1/(s+\lambda)$

Generation of equations:

- Vary $i = 0,1,2 \ldots$ with fixed time horizon
- For fixed minimal i (1 on this case) vary time horizon
- Combination of the above

Integral Equation Approach

PMF transformation with λ=0

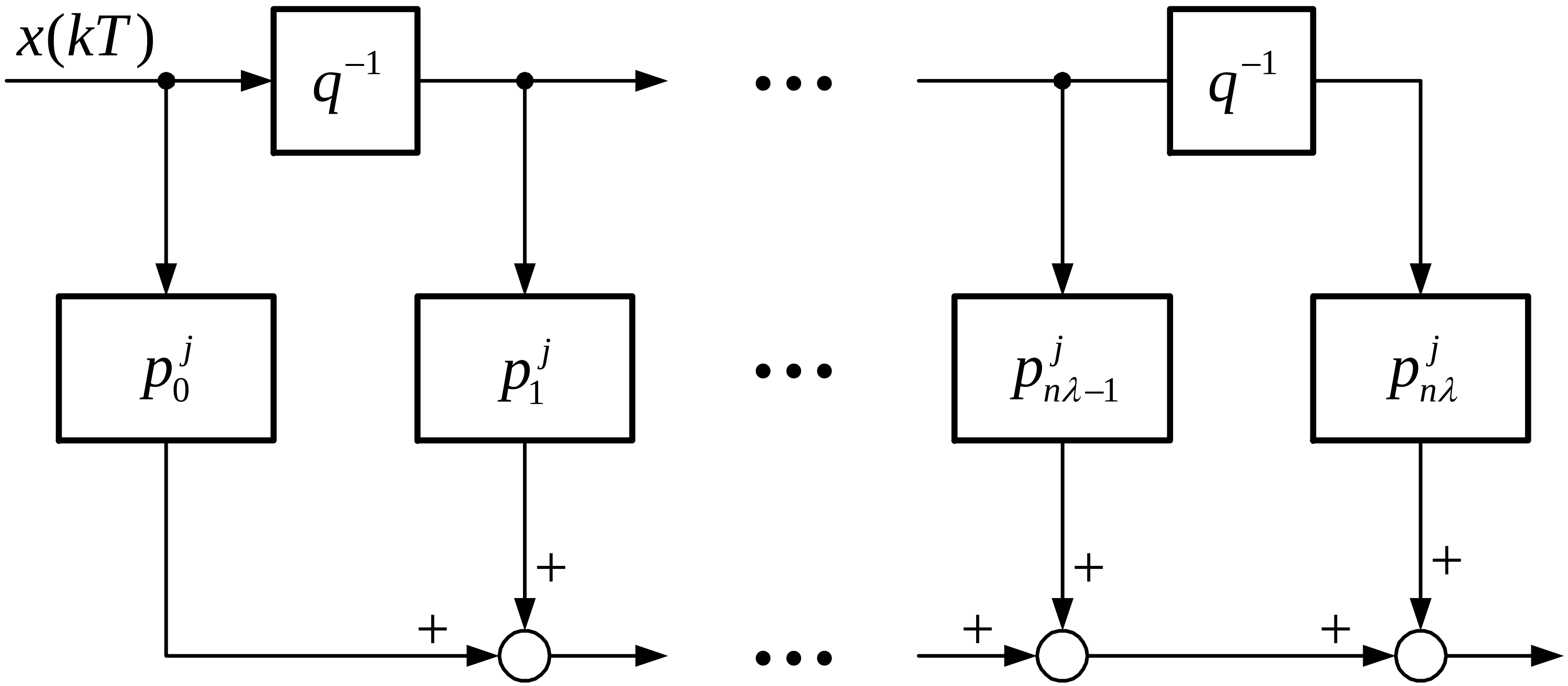


Fig. 6. The linear integrating filter

The Orthogonal Functions (OF) Approach

$$a\mathrm{d}y(t)/\mathrm{d}t + y(t) = bu(t)$$

$$ay(t) - ay(0)s(t) + \int_0^t y(\tau)d\tau = b\int_0^t u(\tau)d\tau,\ 0 \le t \le t_0$$

$$y(t) \approx y_1\theta_1(t) + y_2\theta_2(t) \qquad u(t) \approx u_1\theta_1(t) + u_2\theta_2(t)$$

$$\int_0^t \theta_1(\tau)d\tau \approx e_{11}\theta_1(t) + e_{12}\theta_2(t) \qquad \int_0^t \theta_2(\tau)d\tau \approx e_{21}\theta_1 1(t) + e_{22}\theta_2(t)$$

$$\begin{bmatrix} y(0)s_1 - y_1 & u_1e_{11} + u_2e_{21} \\ y(0)s_2 - y_2 & u_1e_{12} + u_2e_{22} \end{bmatrix} \qquad \begin{bmatrix} y_1e_{11} + y_2e_{21} \\ y_1e_{12} + y_2e_{22} \end{bmatrix}$$

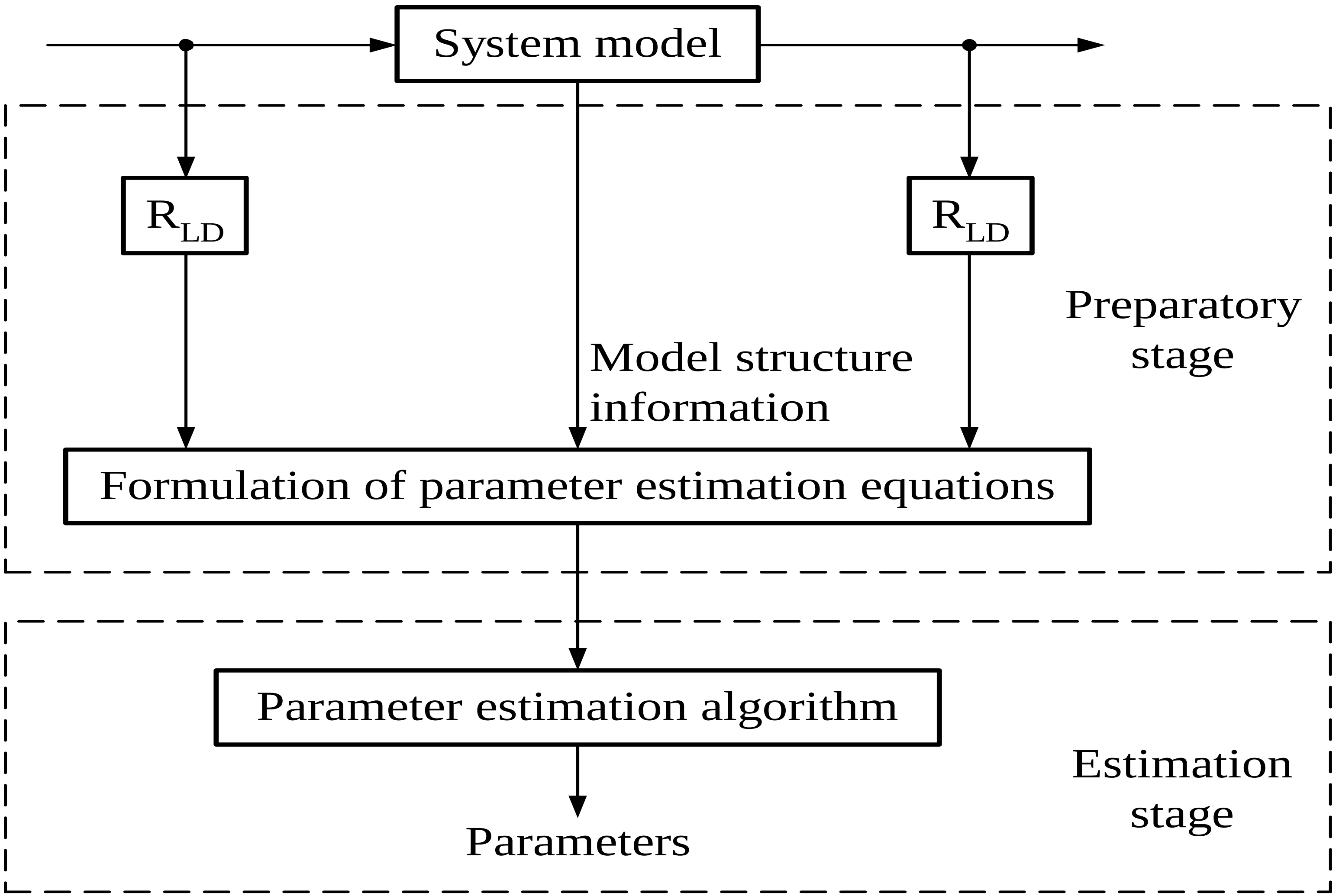


Fig. 7. The general framework for identification of continuous systems.

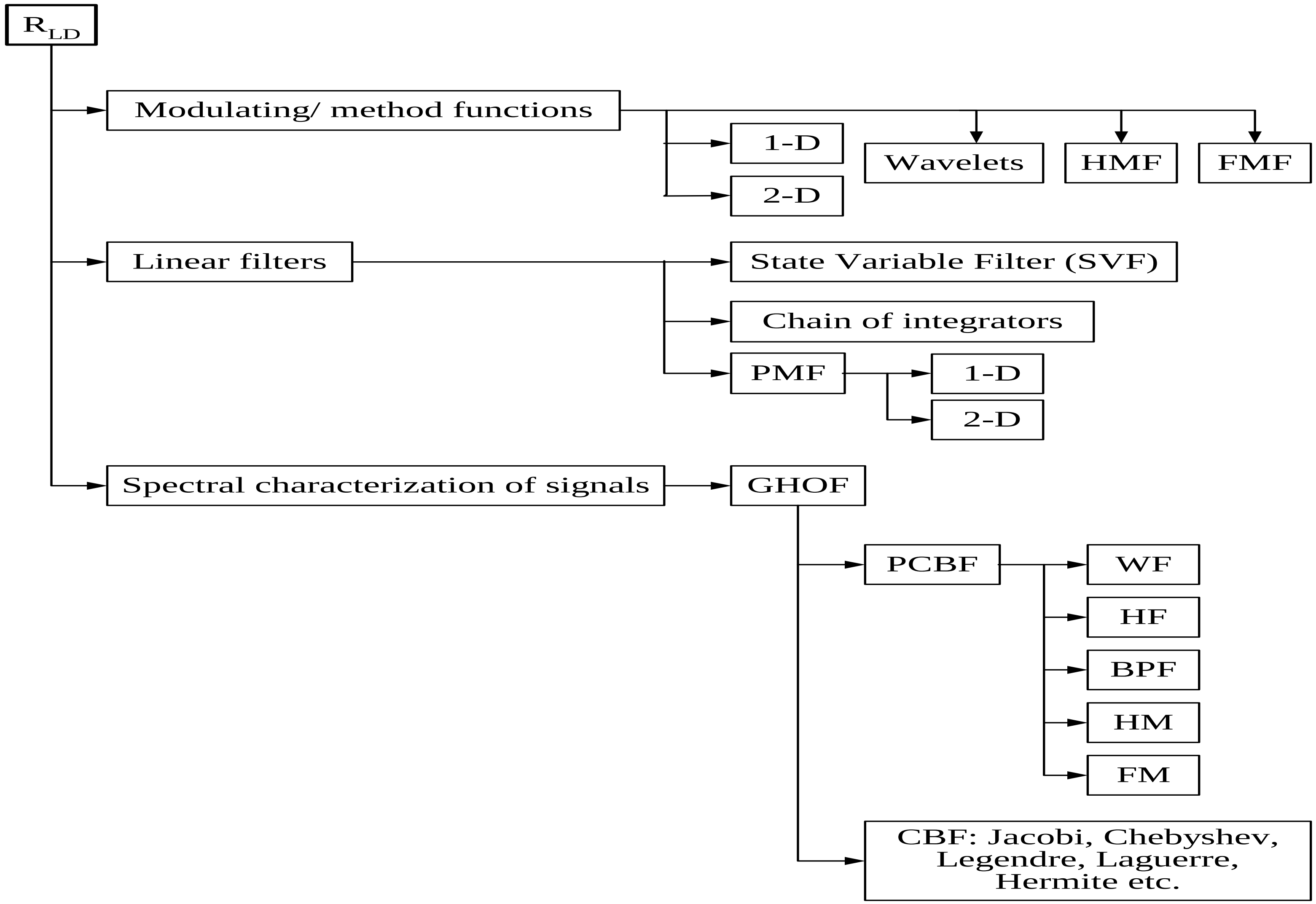


Fig. 8. Several variants of the signal preprocessing operation RLD (PMF: Poisson Moment Functionals; GHOF: General Hybrid Orthogonal Functions; PCBF: Piecewise Constant Basis Functions; CBF: Continuous Basis Functions; WF: Walsh Functions; HF: Haar Functions; BPF: Block Pulse Functions; HTM: Hartley Modulating Functions; FTM: Fourier Modulating Functions).

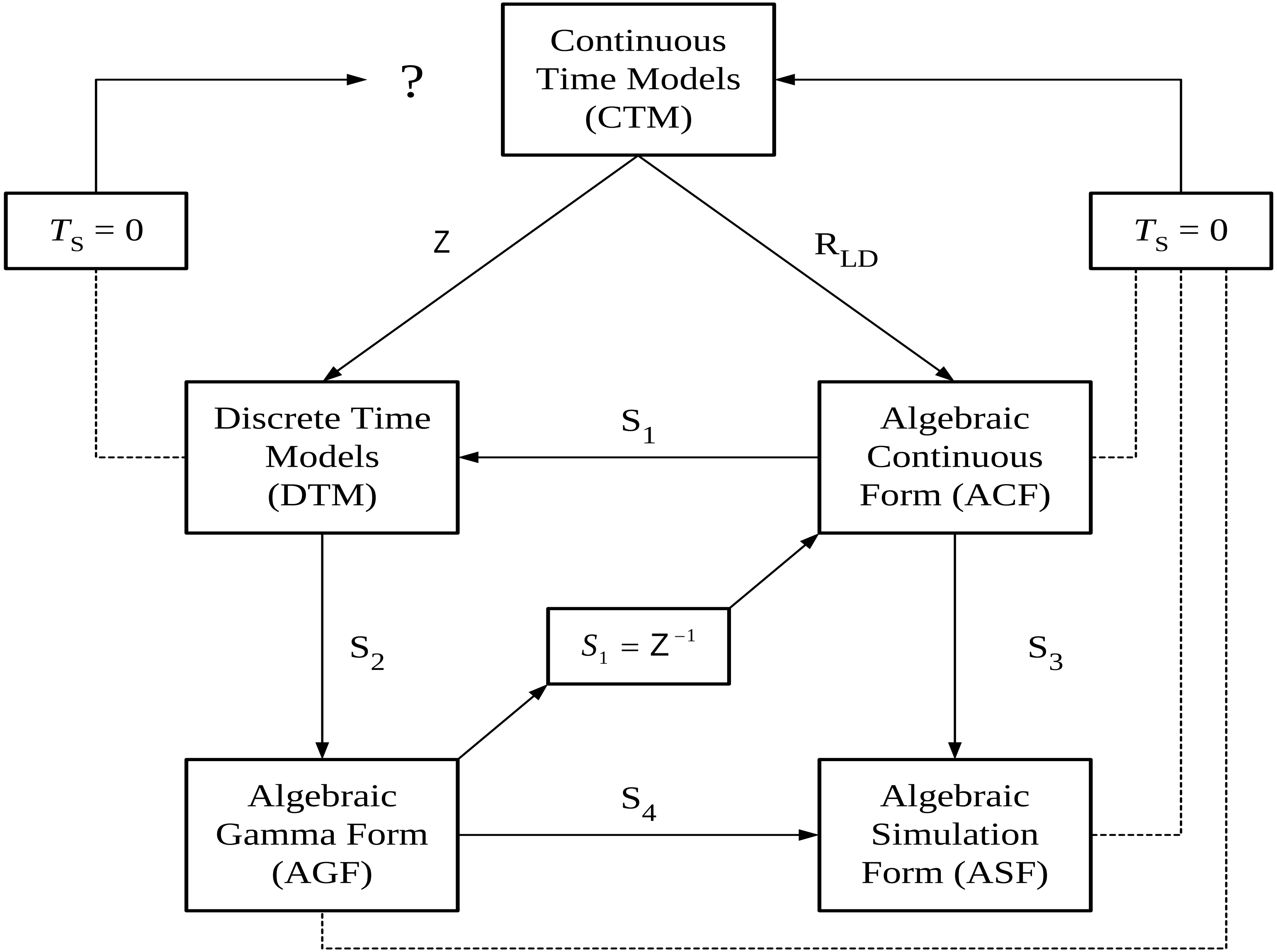
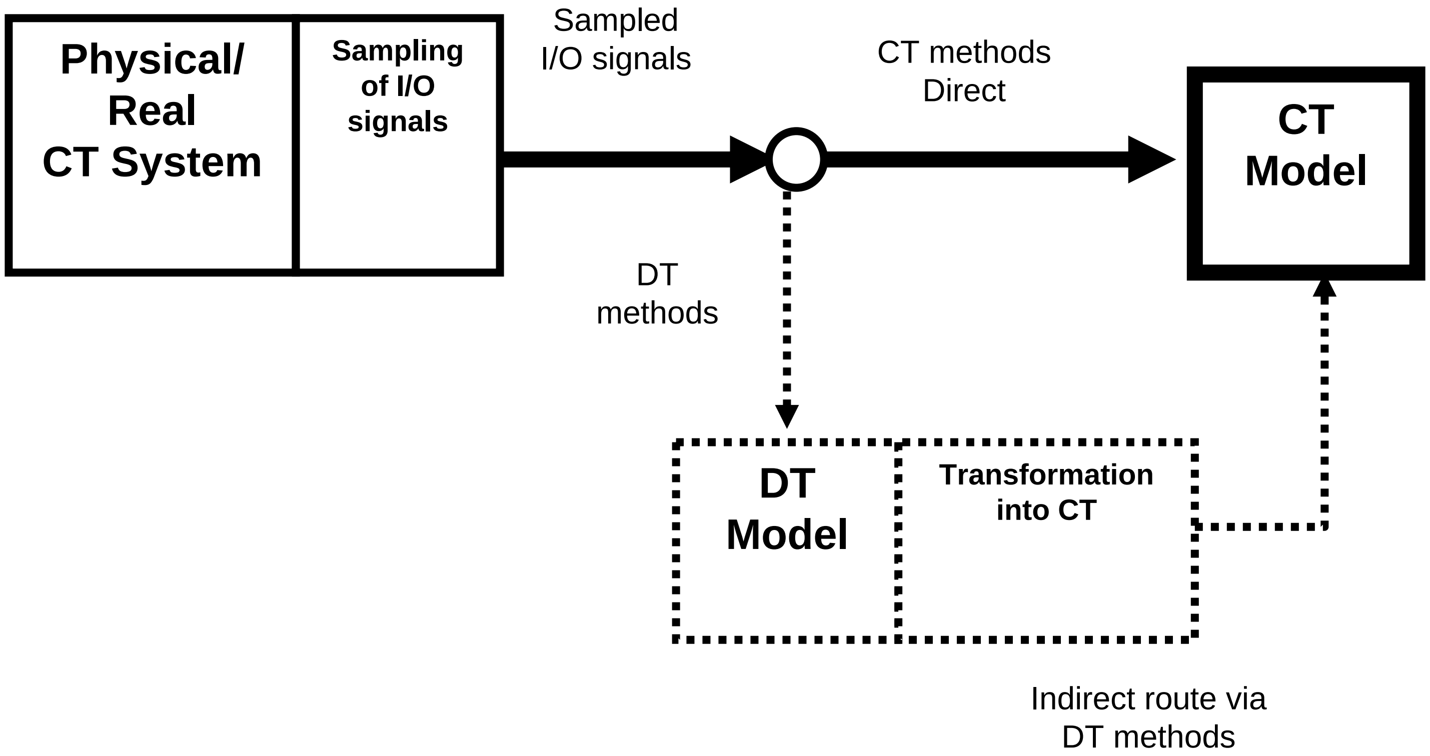


Fig. 9. Reduction of the calculus of continuous time systems into algebra

Secondary Stage

- Apply the standard methods such as Instrumental Variables, etc. to estimate the parameters from the system of equations generated by the Primary Stage

- **indirect approach**: 2 stages: 1. DT model for the original CT system using well-established DT methods and 2. DT model is transformed into CT form
- **direct approach**: CT model is obtained straightaway using well-known CT methods. Matlab CONTSID toolbox (which can be freely downloaded at: *http://www.cran.uhp-nancy.fr*)


Physical/ Real CT System
Sampling of I/O signals
Sampled I/O signals
CT methods Direct
CT Model
DT methods
DT Model
Transformation into CT
Indirect route via DT methods

Problem statement

- SISO system

$$y_u(t) = G_o(p)u(t)$$

$$G_o(p) = \frac{B_o(p)}{A_o(p)}$$

$$B_o(p) = b_0^o + b_1^o p + \cdots + b_m^o p^m$$

$$A_o(p) = a_0^o + a_1^o p + \cdots + a_{n-1}^o p^{n-1} + p^n$$

$$a_0^o y_u(t) + a_1^o y_u^{(1)}(t) + \cdots + y_u^{(n)}(t) = b_0^o u(t) + b_1^o u^{(1)}(t) + \cdots + b_m^o u^{(m)}(t)$$

$$y(t) = G_o(p)u(t) + v_o(t)$$

$$\theta = \left[a_{n-1}^o \ldots a_0^o \; b_m^o \ldots b_0^o \right]^T$$

$$Z^N = \left[u(t_k); y(t_k) \right]_{k=1}^N$$

Assessment of identification algorithms

- Success/failure

$$\eta = \frac{1 - F}{N_{\exp}} = \frac{S}{N_{\exp}}$$

Accuracy

- Measures

$$\bar{\varepsilon}_{\hat{y}_u} = \frac{1}{N_{\exp}} \sum_{i=1}^{N_{\exp}} \varepsilon_{\hat{y}_u}(i)$$

$$\sigma_{\hat{y}_u}^2 = \frac{1}{N_{\exp}} \sum_{i=1}^{N_{\exp}} \left(\varepsilon_{\hat{y}_u}(i) - \bar{\varepsilon}_{\hat{y}_u} \right)^2$$

$$\varepsilon_{\hat{y}_u}(i) = \frac{1}{N} \sqrt{\sum_{i=1}^{N} \left(y_{u,i}(t_k) - \hat{y}_{u,i}(t_k) \right)^2}$$

Frequency response

- Measures

$$\bar{\varepsilon}_{\hat{G}} = \frac{1}{N_{\exp}} \sum_{i=1}^{N_{\exp}} \sum_{j=1}^{N_{\omega}} \left(G_o(\omega_j) - \hat{G}_i(\omega_j) \right)^2$$

$$\bar{\varepsilon}_{\hat{\phi}} = \frac{1}{N_{\exp}} \sum_{i=1}^{N_{\exp}} \sum_{j=1}^{N_{\omega}} \left(\phi_o(\omega_j) - \hat{\phi}_i(\omega_j) \right)^2$$

Parameters

- Parameter error

$$\delta = \left\| \theta_o - \hat{\theta} \right\|$$

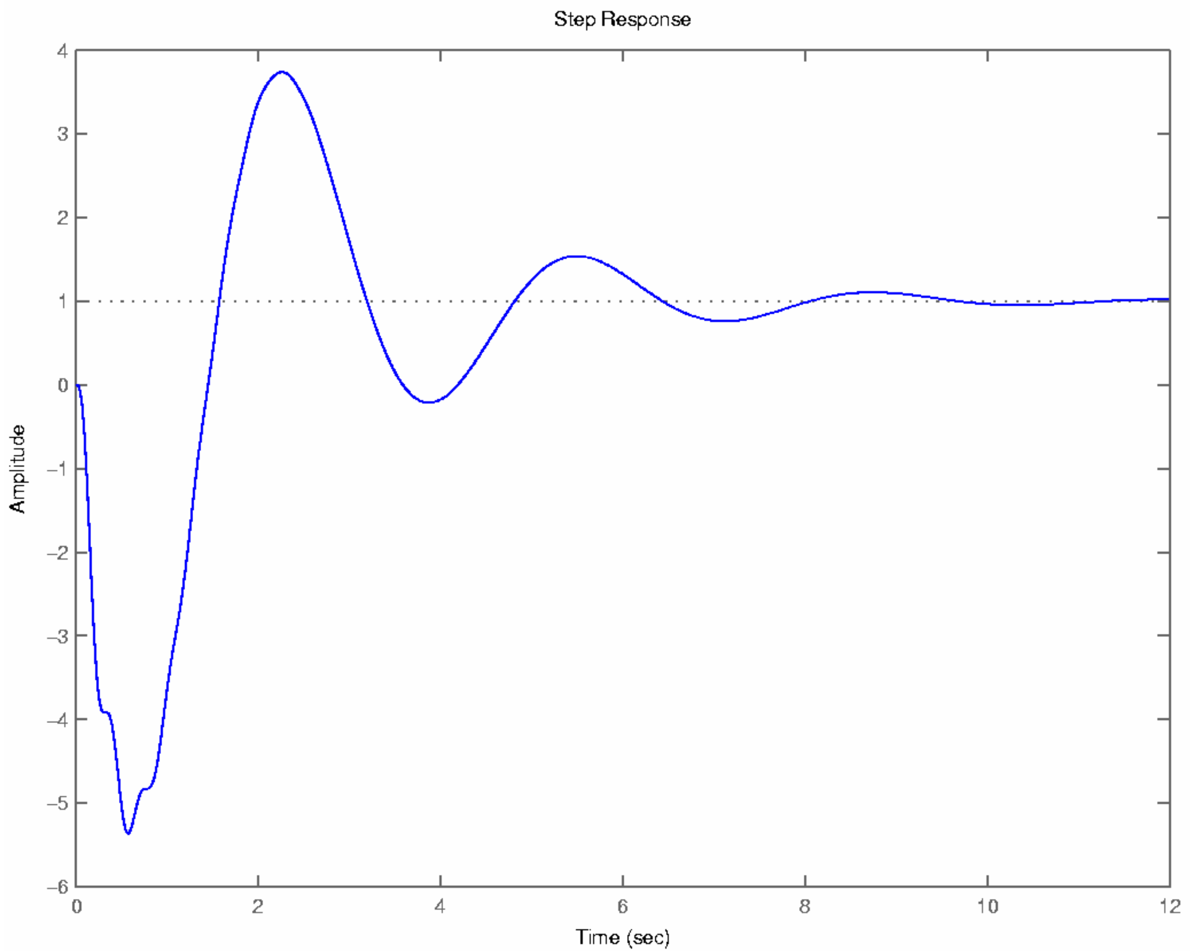
$$NMSE(\hat{\theta}_j) = \frac{1}{N_{\exp}} \sum_{i=1}^{N_{\exp}} \left(\frac{\theta_o^j - \hat{\theta}^j(i)}{\theta_o^j} \right)^2$$

Rao-Garnier test system (Ljung 1993)

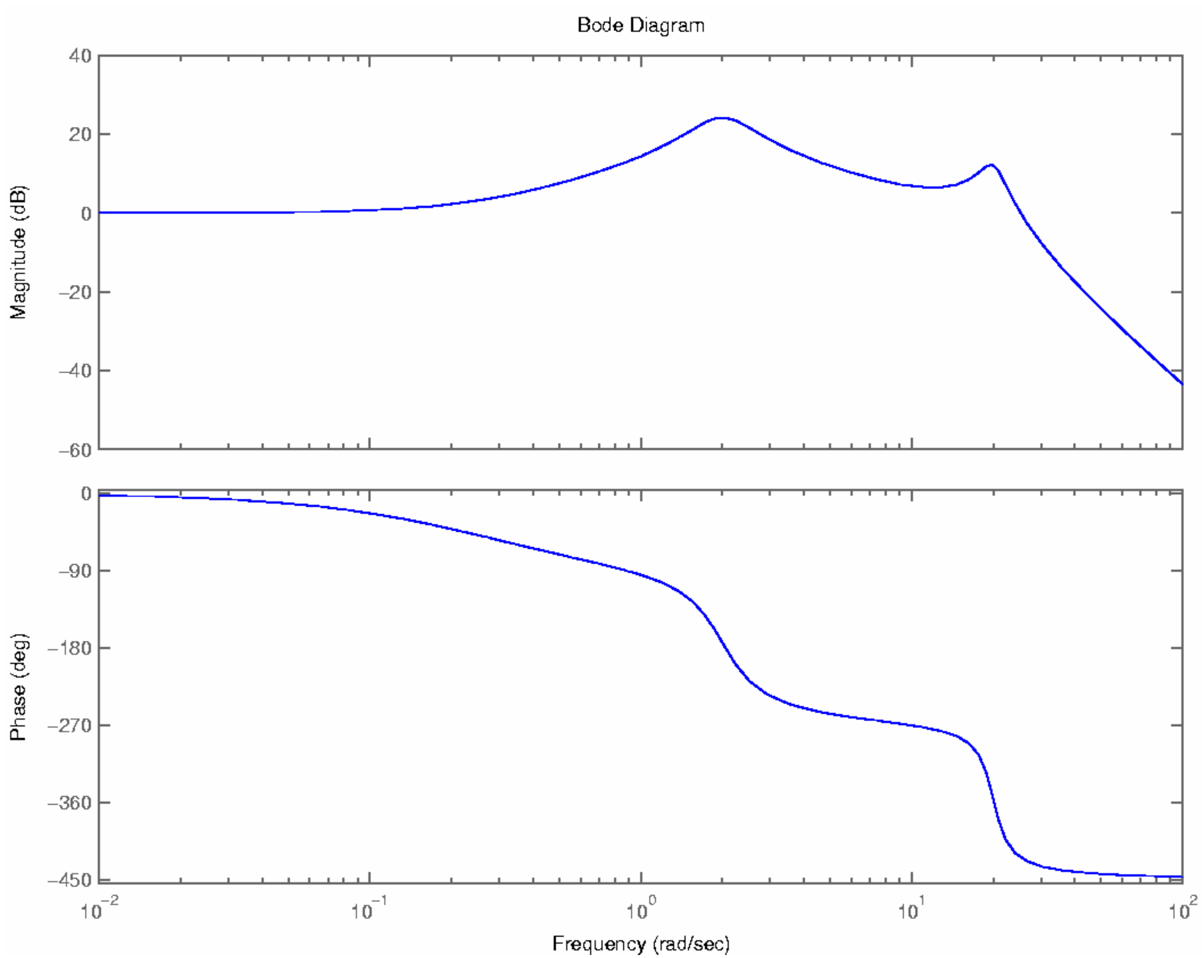
- The system

$$G_o(s) = \frac{K(-Ts+1)}{\left(\frac{s^2}{\omega_{n,1}^2} + \frac{2\zeta_1 s}{w_{n,1}} + 1\right)\left(\frac{s^2}{\omega_{n,2}^2} + \frac{2\zeta_2 s}{w_{n,2}} + 1\right)}$$

Step response of RGTS



FR of RGTS


Bode Diagram
Magnitude (dB)
40
20
0
−20
−40
−60
Phase (deg)
0
−90
−180
−270
−360
−450
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/sec)

$u(t)=\sin(t)+\sin(1.9t)+\sin(2.1t)+\sin(18t)+\sin(22)$
The observation time is set to T=75s. Because of the two sampling periods, the input signal has 1500 or 7500 samples.

PRBS of maximum length. The characteristics of the signal, whose amplitude switches between -1 and +1, are the following: the number of stages of the shift register is set to ns=10, the clock period is set to np=10, which makes a number of points N=7161 for a sampling period setting of 10ms. With ns=9 and np=3 we have N=1533 for a sampling period setting of 50ms.

Types of noise

- The following types of measurement noise *v*(*tk*) are considered
- (a) *DT white noise : v*(*tk*)= *e*(*tk*)

  (b) DT colored noise (ARMA process noise)

$$v(t_k) = \frac{0.2236q^{-1} - 0.1630q^{-2}}{1 - 1.8906q^{-1} + 0.9512q^{-2}} e(t_k)$$

Simulation conditions					SID toolbox methods				CONTSID toolbox methods					Bode plot
T_s	input	noise	name	criterion	IV4	N4SID	OE	PEM	IVFMF	IVLIF	IVGPMF	COE	SRIVC	
50ms	PRBS	noise-free	trial10	SE	3.9e-18	1.7e-25	2.5e-26	2.6e-26	3.1	9.2e-1	1.1e-2	1.3e-9	1.6e-9	
50ms	multi-sine	noise-free	trial3	SE	1.6e+2	2.1e-2	1.2e-4	7.8e-3	2.2	1.7	2.3	2.4	7.9e-1	
50ms	PRBS	white 10dB	trial11	η_i	48	100	94	92	100	100	100	100	100	Figures (2) and (3)
				MSE	2.6e+1	5.8	2.4	1.1e+1	4.4	1.0	4.5e-2	8.4e-3	8.4e-3	
				σ_{SE}	2.8e+1	1.2e+1	4.8	8.1	1.6	2.1e-1	2.3e-2	4.8e-3	4.8e-3	
				$E_{\hat{G}_i}$	2.09e+4	2.95e+3	2.67e+3	1.15e+4	4.10e+3	7.92e+2	8.00e+1	1.48e+1	1.48e+1	
				$E_{\hat{\phi}_i}$	1.64e+7	2.22e+5	3.64e+6	1.19e+7	9.63e+5	3.38e+5	2.20e+3	3.93e+2	3.93e+2	
50ms	multi-sine	white 10dB	trial4	η_i	49	69	97	99	100	100	100	100	100	Figures (4) and (5)
				MSE	4.8e+1	1.0e+1	2.5	6.4e+1	2.5	2.2	2.2	2.5	9.1e-1	
				σ_{SE}	2.3e+1	1.7e+1	1.1e+1	6.2e+1	5.9e-1	7.5e-1	3.5e-1	3.1e-1	6.4e-2	
				$E_{\hat{G}_i}$	6.69e+4	2.79e+4	1.16e+4	6.16e+4	1.49e+2	3.55e+2	5.21e+1	4.07e+1	2.34e+2	
				$E_{\hat{\phi}_i}$	9.50e+5	3.32e+5	4.38e+5	1.66e+6	3.94e+3	1.13e+4	1.34e+3	9.38e+2	8.48e+3	

Table 1. Monte Carlo simulation results when $T_s = 50$ ms

Simulation conditions					SID toolbox methods				CONTSID toolbox methods					Bode plot
T_s	input	noise	name	criterion	IV4	N4SID	OE	PEM	IVFMF	IVLIF	IVGPMF	COE	SRIVC	
10ms	PRBS	noise-free	trial8	SE	1.1e-3	1.9e-018	1.8e-20	8.8e-10	1.1e-1	4.3e-2	1.1e-5	3.4e-12	3.7e-10	
10ms	multi-sine	noise-free	trial1	SE	1.6e+3	1.8e-6	6.0e-9	9.6e-6	1.1e-1	8.4e-2	1.1e-1	1.1e-1	4.0e-2	
10ms	PRBS	white 10dB	trial9	η_i	33	63	92	92	100	100	100	100	100	Figures (6) and (7)
				MSE	7.7	1.0e+1	1.4	7.7	1.7e-1	4.7e-2	3.0e-3	8.4e-4	8.4e-4	
				σ_{SE}	2.4	6.3	2.6	2.6	1.0e-1	1.6e-2	2.0e-3	4.5e-4	4.5e-4	
				$E_{\hat{G}_i}$	5.60e+4	4.21e+4	1.64e+4	4.31e+4	5.61e+2	3.78e+2	1.21e+1	2.41e+0	2.41e+0	
				$E_{\hat{\phi}_i}$	1.88e+7	1.80e+7	3.55e+6	1.77e+7	1.82e+5	1.70e+4	2.89e+2	5.66e+1	5.66e+1	
10ms	multi-sine	white 10dB	trial2	η_i	13	52	97	99	100	100	100	100	100	Figures (8) and (9)
				MSE	1.3e+2	3.1e+3	1.7e+1	6.9e+4	1.9e-1	1.2e-1	1.4e-1	1.3e-1	6.3e-2	
				σ_{SE}	1.3e+2	1.3e+4	4.7e+1	9.7e+5	8.3e-2	3.9e-2	3.8e-2	3.7e-2	1.3e-2	
				$E_{\hat{G}_i}$	3.98e+5	4.38e+5	1.76e+5	3.56e+5	2.93e+1	1.23e+1	7.99e+0	7.37e+0	1.31e+1	
				$E_{\hat{\phi}_i}$	2.10e+6	6.69e+5	2.22e+6	7.42e+6	6.23e+2	2.65e+2	1.70e+2	1.64e+2	3.01e+2	
10ms	multi-sine	white 0dB	trial5	η_i	16	60	98	99	100	100	100	100	100	Figures (??) and (??)
				MSE	1.7e+2	1.1e+3	1.4e+9	1.2e+5	1.0	2.1	4.2e-1	3.4e-1	2.7e-1	
				σ_{SE}	9.7e+1	2.0e+3	2.010	1.7e+6	5.5e-1	3.5	2.0e-1	1.6e-1	1.3e-1	
				$E_{\hat{G}_i}$	4.28e+5	6.28e+5	3.75e+5	3.85e+5	2.92e+2	9.96e+2	9.09e+1	7.26e+1	7.71e+1	
				$E_{\hat{\phi}_i}$	1.89e+7	1.16e+6	1.47e+7	9.60e+6	6.36e+3	8.58e+3	2.09e+3	1.79e+3	1.84e+3	
10ms	multi-sine	colored 10dB	trial7	η_i	20	80	100	100	100	100	100	100	100	Figures (??) and (??)
				MSE	9.0e+1	1.2e+2	2.9	5.4e+2	3.3e-1	2.9e-1	3.2e-1	2.1e-1	1.7e-1	
				σ_{SE}	1.6e+1	2.2e+1	2.2e+1	7.5e+3	2.5e-1	1.8e-1	2.3e-1	1.1e-1	1.2e-1	
				$E_{\hat{G}_i}$	2.72e+5	2.93e+5	7.75e+4	1.21e+5	4.14e+1	6.66e+1	6.12e+1	2.10e+1	3.04e+1	
				$E_{\hat{\phi}_i}$	5.37e+6	1.96e+6	6.92e+5	6.95e+5	9.46e+2	7.81e+2	7.77e+2	4.36e+2	6.86e+2	
10ms	multi-sine	colored 0dB	trial6	η_i	29	81	98	99	93	92	96	100	100	Figures (??) and (??)
				MSE	1.1e+2	1.2e+2	6.5	1.4e+1	1.5e+1	1.2e+1	6.0	1.9	1.5	
				σ_{SE}	8.3e+1	1.5e+1	2.9e+1	3.7e+1	9.9e+1	2.7e+1	2.2e+1	6.2	1.4	
				$E_{\hat{G}_i}$	3.27e+5	3.17e+5	1.01e+5	1.96e+5	2.49e+3	4.65e+3	1.68e+3	5.76e+2	5.67e+2	
				$E_{\hat{\phi}_i}$	5.39e+6	3.40e+6	8.40e+5	1.55e+6	2.81e+4	8.00e+4	1.53e+4	1.48e+4	1.04e+4	

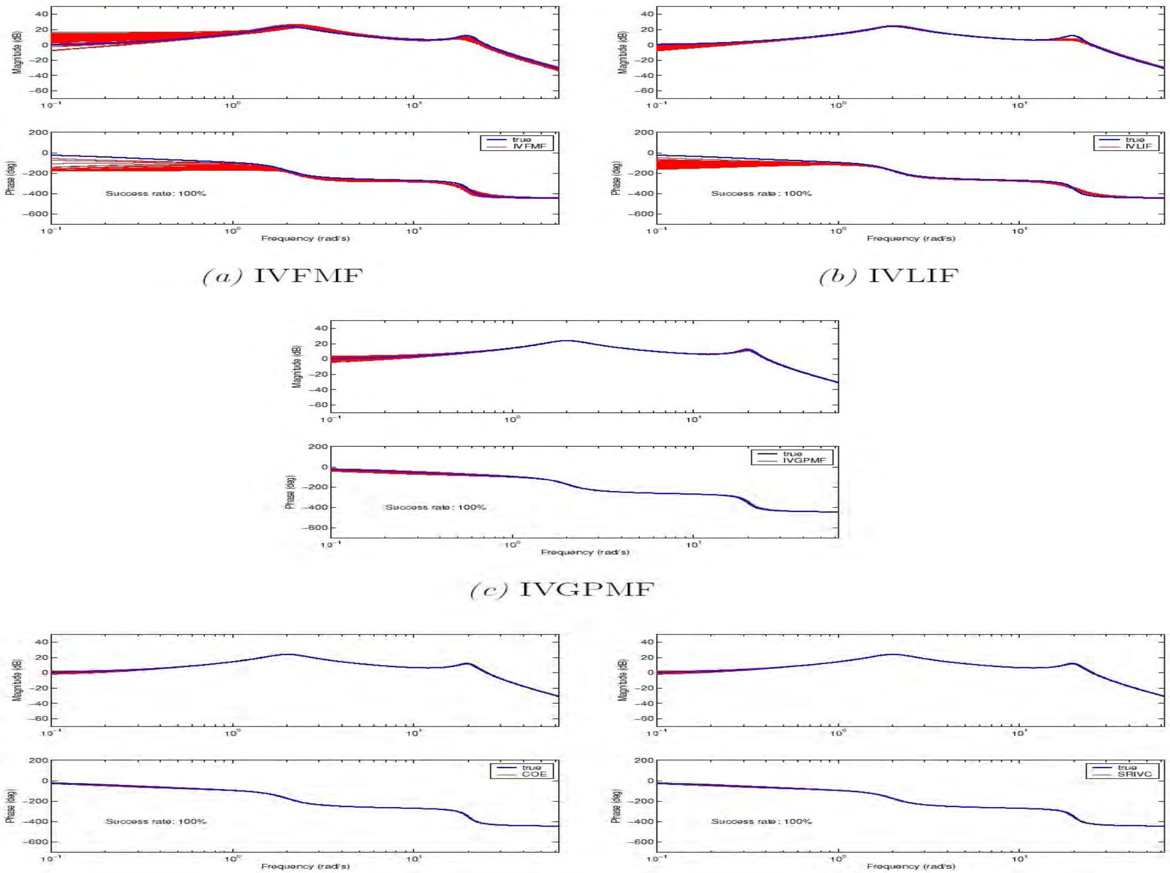
Table 2. Monte Carlo simulation results when $T_s = 10$ ms

Simulation conditions					Prefiltering				Decimation				Bode plot
T_s	input	noise	name	criterion	IV4	N4SID	OE	PEM	IV4	N4SID	OE	PEM	
50ms	PRBS	white 10dB	trial11	η_i	100	100	100	100	100	100	100	100	Figures (??) and (??)
				MSE	6.8e-1	4.0e-1	1.1e-2	1.1e-2	7.6e-1	9.6e-1	7.5e-1	7.6e-1	
				σ_{SE}	1.3e-1	4.7e-1	5.6e-3	5.6e-3	7.5e-2	9.6e-2	7.2e-2	7.2e-2	
				$E_{\hat{G}_i}$	1.04e+3	1.60e+3	2.08e+2	2.08e+2	1.28e+2	2.62e+2	1.16e+2	1.13e+2	
				$E_{\hat{\phi}_i}$	1.60e+5	8.98e+4	2.09e+3	2.09e+3	3.52e+4	3.60e+4	3.37e+4	3.50e+4	
50ms	multi-sine	white 10dB	trial4	η_i	100	95	92	92	100	100	100	100	Figures (??) and (??)
				MSE	4.1	4.4	2.1	2.1	2.8	6.6	2.7	5.6	
				σ_{SE}	7.2e-1	2.5	2.0e+1	2.0e+1	5.2e-1	8.6e-1	4.8e-1	1.7e+1	
				$E_{\hat{G}_i}$	3.56e+3	1.10e+4	7.91e+2	7.91e+2	1.36e+2	2.69e+3	1.06e+2	5.84e+2	
				$E_{\hat{\phi}_i}$	7.05e+5	5.42e+5	3.16e+5	3.16e+5	1.31e+5	9.60e+4	1.34e+5	1.56e+5	
10ms	PRBS	white 10dB	trial9	η_i	100	97	100	100	100	100	100	100	Figures (??) and (??)
				MSE	1.2e-2	5.8	1.3e-3	1.3e-3	1.8	1.9	1.8	1.8	
				σ_{SE}	3.3e-3	8.7e-1	6.9e-4	6.9e-4	4.6e-2	4.8e-2	4.4e-2	4.5e-2	
				$E_{\hat{G}_i}$	4.17e+3	4.07e+4	2.30e+3	2.30e+3	1.47e+2	1.78e+2	1.38e+2	1.44e+2	
				$E_{\hat{\phi}_i}$	6.52e+5	1.61e+7	2.02e+5	2.02e+5	1.08e+5	1.07e+5	1.06e+5	1.07e+5	
10ms	multi-sine	white 0dB	trial2	η_i	100	100	95	95	100	100	100	100	Figures (??) and (??)
				MSE	3.3	1.1e+1	4.4	4.4	9.4	1.6e+1	9.4	9.4	
				σ_{SE}	4.7e-1	2.1e+1	3.4e+1	3.4e+1	4.4e-1	3.2	4.3e-1	4.4e-1	
				$E_{\hat{G}_i}$	4.15e+4	4.64e+4	6.68e+3	6.68e+3	1.52e+2	4.05e+3	1.36e+2	1.62e+2	
				$E_{\hat{\phi}_i}$	1.73e+6	1.61e+6	6.43e+5	6.43e+5	1.29e+5	1.09e+5	1.31e+5	1.28e+5	

Table 3. Monte Carlo simulation results obtained in the case of DT methods from pre-filtered and decimated data (The criteria presented in this table for the DT methods have been computed from the CT version of the estimated DT models)

		b_3	b_2	b_1	b_0	a_3	a_2	a_1	a_0	$\sum_{j=1}^{6} NMSE(\hat{\theta}_j)$
method	True value	0	0	-6400	1600	5	408	416	1600	
IV4 (focus)	$\overline{\hat{\theta}_j}$	-11.0	72.7	-10424.9	1578.8	4.6	456.1	655.9	1926.6	
	$\sigma_{\hat{\theta}_j}$	1.0	12.7	395.3	153.3	0.1	5.3	23.8	30.9	
	NMSE($\hat{\theta}_j$)			4.0e-1	9.3e-3	5.6e-3	1.4e-2	3.4e-1	4.2e-2	0.8062
N4SID (focus)	$\overline{\hat{\theta}_j}$	1.6	-160.7	-8239.2	568.1	9.0	461.1	550.3	1892.4	
	$\sigma_{\hat{\theta}_j}$	31.3	1324.0	14837.9	2152.3	24.4	361.6	1036.3	2178.6	
	NMSE($\hat{\theta}_j$)			5.4	2.2	2.4e+1	8.0e-1	6.3	1.9	40.9996
OE (focus)	$\overline{\hat{\theta}_j}$	2.7	-49.4	-6335.8	1589.2	6.5	411.3	428.5	1585.5	
	$\sigma_{\hat{\theta}_j}$	20.7	219.0	1759.5	621.8	11.0	88.6	112.5	430.4	
	NMSE($\hat{\theta}_j$)			7.5e-2	1.5e-1	4.9	4.7e-2	7.4e-2	7.2e-2	5.3568
PEM (focus)	$\overline{\hat{\theta}_j}$	2.7	-49.4	-6335.8	1589.2	6.5	411.3	428.5	1585.5	
	$\sigma_{\hat{\theta}_j}$	20.7	219.0	1759.5	621.8	11.0	88.6	112.5	430.4	
	NMSE($\hat{\theta}_j$)			7.5e-2	1.5e-1	4.9	4.7e-2	7.4e-2	7.2e-2	5.3568
IV4 (dec)	$\overline{\hat{\theta}_j}$	-4.7	-309.5	-6315.4	1645.7	5.0	412.3	416.3	1616.7	
	$\sigma_{\hat{\theta}_j}$	0.9	13.9	138.9	102.2	0.2	3.8	8.4	16.6	
	NMSE($\hat{\theta}_j$)			6.4e-4	4.9e-3	2.1e-3	2.0e-4	4.0e-4	2.2e-4	0.0085
N4SID (dec)	$\overline{\hat{\theta}_j}$	30.4	28.3	-11541.1	3339.6	7.4	724.9	736.2	2981.1	
	$\sigma_{\hat{\theta}_j}$	16.3	51.4	812.2	503.1	0.9	46.5	52.5	200.7	
	NMSE($\hat{\theta}_j$)			6.6e-1	1.3	2.6e-1	6.2e-1	6.1e-1	7.6e-1	4.1919
OE (dec)	$\overline{\hat{\theta}_j}$	-5.2	-305.7	-6390.5	1648.5	4.9	411.6	420.1	1617.1	
	$\sigma_{\hat{\theta}_j}$	0.7	12.8	113.4	98.8	0.2	3.7	7.2	17.3	
	NMSE($\hat{\theta}_j$)			3.1e-4	4.7e-3	1.8e-3	1.6e-4	4.0e-4	2.3e-4	0.0077
PEM (dec)	$\overline{\hat{\theta}_j}$	-4.7	-309.7	-6397.9	1651.2	5.0	413.2	421.1	1622.6	
	$\sigma_{\hat{\theta}_j}$	1.3	14.7	123.4	105.9	0.3	5.6	8.2	23.1	
	NMSE($\hat{\theta}_j$)			3.7e-4	5.4e-3	2.9e-3	3.5e-4	5.4e-4	4.1e-4	0.0100
IVFMF	$\overline{\hat{\theta}_j}$			-6395.9	1584.1	5.0	407.8	415.6	1600.1	
	$\sigma_{\hat{\theta}_j}$			223.3	183.8	0.3	5.9	14.4	28.3	
	NMSE($\hat{\theta}_j$)			1.2e-3	1.3e-2	3.7e-3	2.1e-4	1.2e-3	3.1e-4	0.0199
IVLIF	$\overline{\hat{\theta}_j}$			-6465.5	1596.1	5.0	410.4	420.1	1611.0	
	$\sigma_{\hat{\theta}_j}$			138.2	116.4	0.2	3.1	8.6	16.0	
	NMSE($\hat{\theta}_j$)			5.7e-4	5.3e-3	1.6e-3	9.1e-5	5.2e-4	1.5e-4	0.0082
IVGPMF	$\overline{\hat{\theta}_j}$			-6397.7	1580.8	5.0	408.0	415.8	1601.0	
	$\sigma_{\hat{\theta}_j}$			96.3	100.0	0.1	2.5	6.3	12.3	
	NMSE($\hat{\theta}_j$)			2.3e-4	4.0e-3	9.0e-4	3.8e-5	2.3e-4	6.0e-5	0.0055
COE	$\overline{\hat{\theta}_j}$			-6400.0	1581.1	5.0	407.9	415.8	1600.7	
	$\sigma_{\hat{\theta}_j}$			84.2	99.5	0.1	2.4	5.6	12.0	
	NMSE($\hat{\theta}_j$)			1.7e-4	4.0e-3	7.5e-4	3.4e-5	1.8e-4	5.6e-5	0.0052
SRIVC	$\overline{\hat{\theta}_j}$			-6555.2	1677.3	5.1	416.2	426.0	1638.0	
	$\sigma_{\hat{\theta}_j}$			89.0	101.7	0.1	2.5	5.9	12.7	
	NMSE($\hat{\theta}_j$)			7.8e-4	6.3e-3	9.4e-4	4.4e-4	7.8e-4	6.3e-4	0.0099

Table 4
Monte Carlo simulation results for the fourth-order system in case of trial2



(a) IVFMF (b) IVLIF

(c) IVGPMF

(d) COE (e) SRIVC

Fig. 2. Bode plots of CT models resulting from direct methods (Trial11)

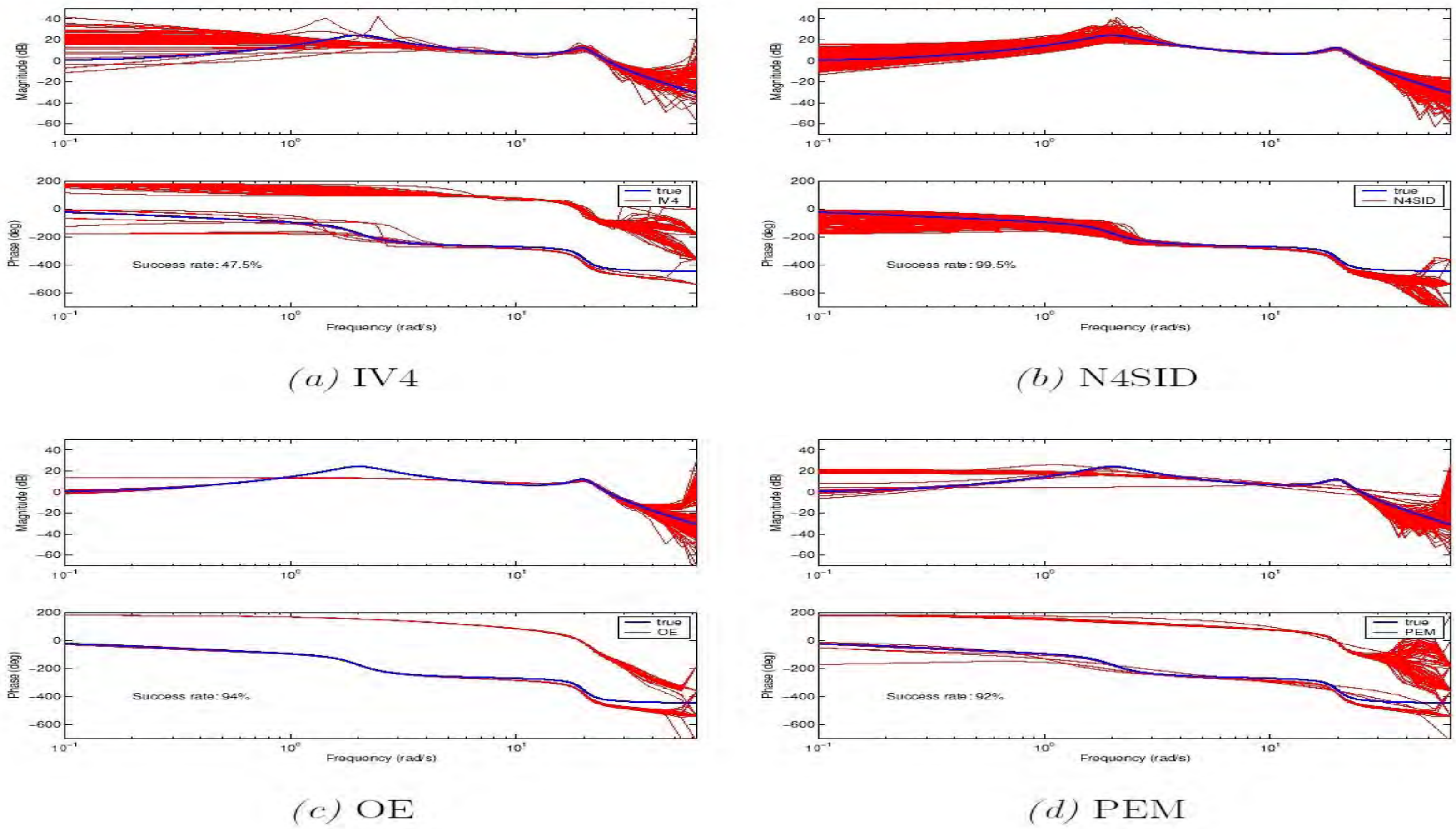
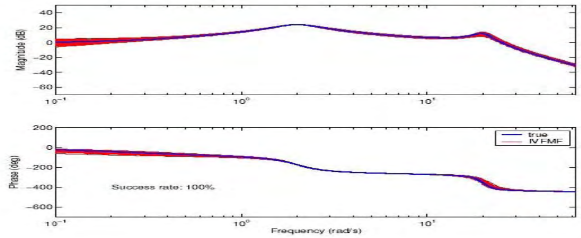
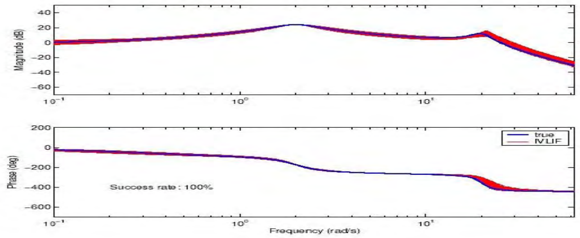


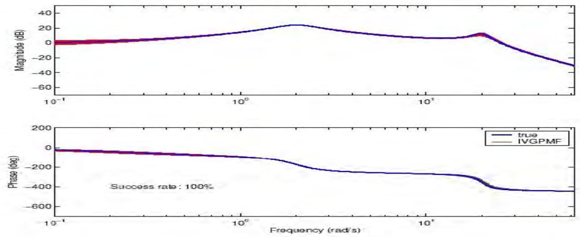
Fig. 3. Bode plots of DT models resulting from DT methods (Trial11)



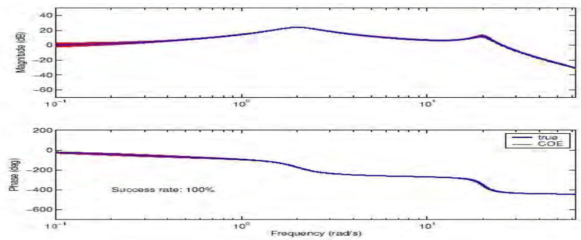
(a) IVFMF



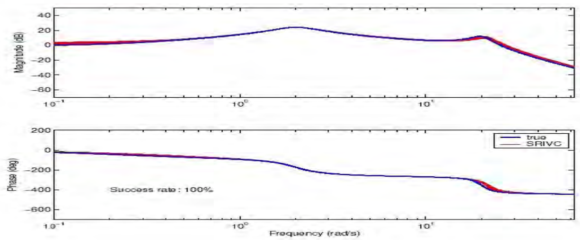
(b) IVLIF



(c) IVGPMF



(d) COE



(e) SRIVC

Fig. 4. Bode plots of CT models resulting from direct methods (Trial4)

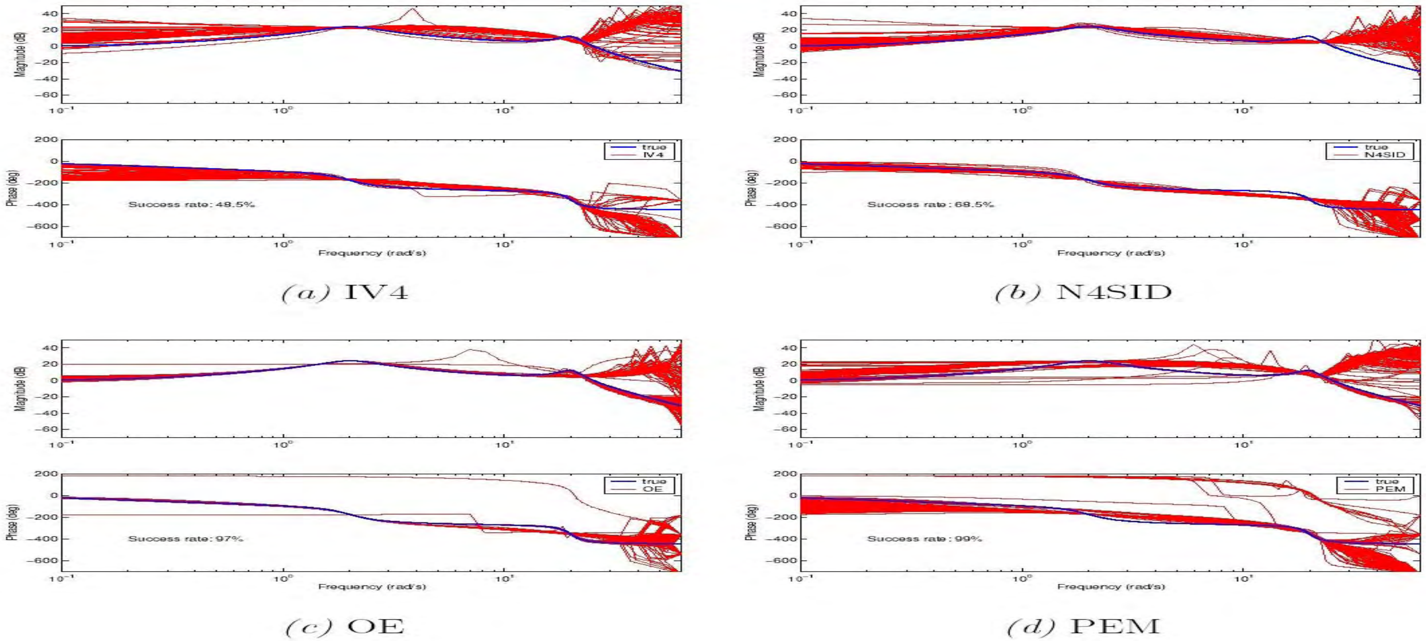
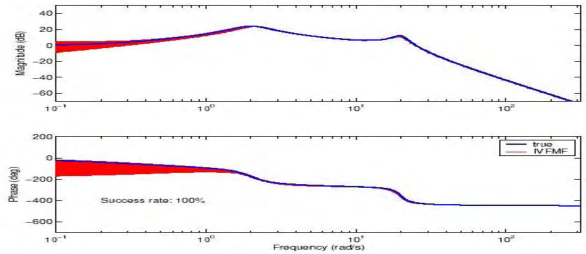
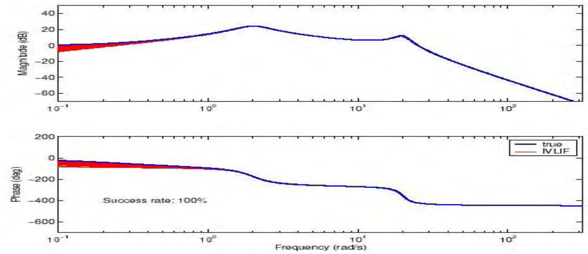


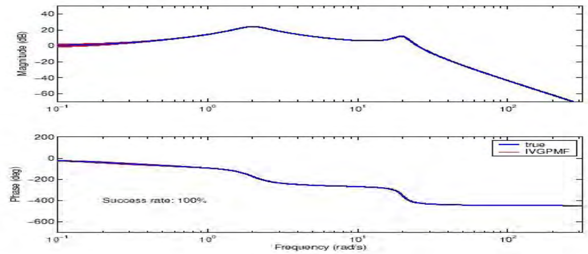
Fig. 5. Bode plots of DT models resulting from DT methods (Trial4)



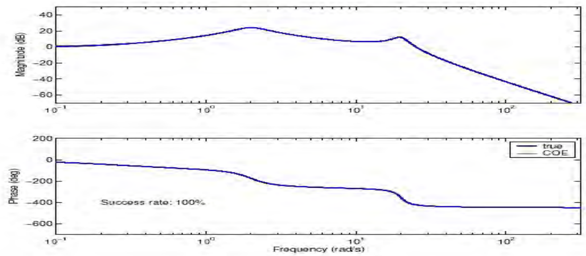
(a) IVFMF



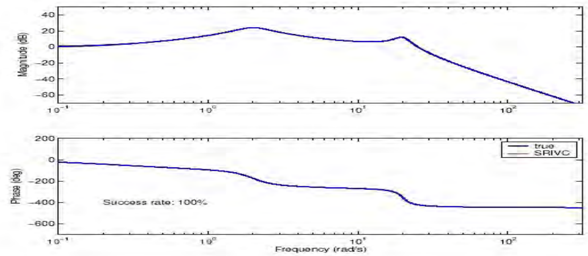
(b) IVLIF



(c) IVGPMF



(d) COE



(e) SRIVC

Fig. 6. Bode plots of CT models resulting from direct methods (Trial9)

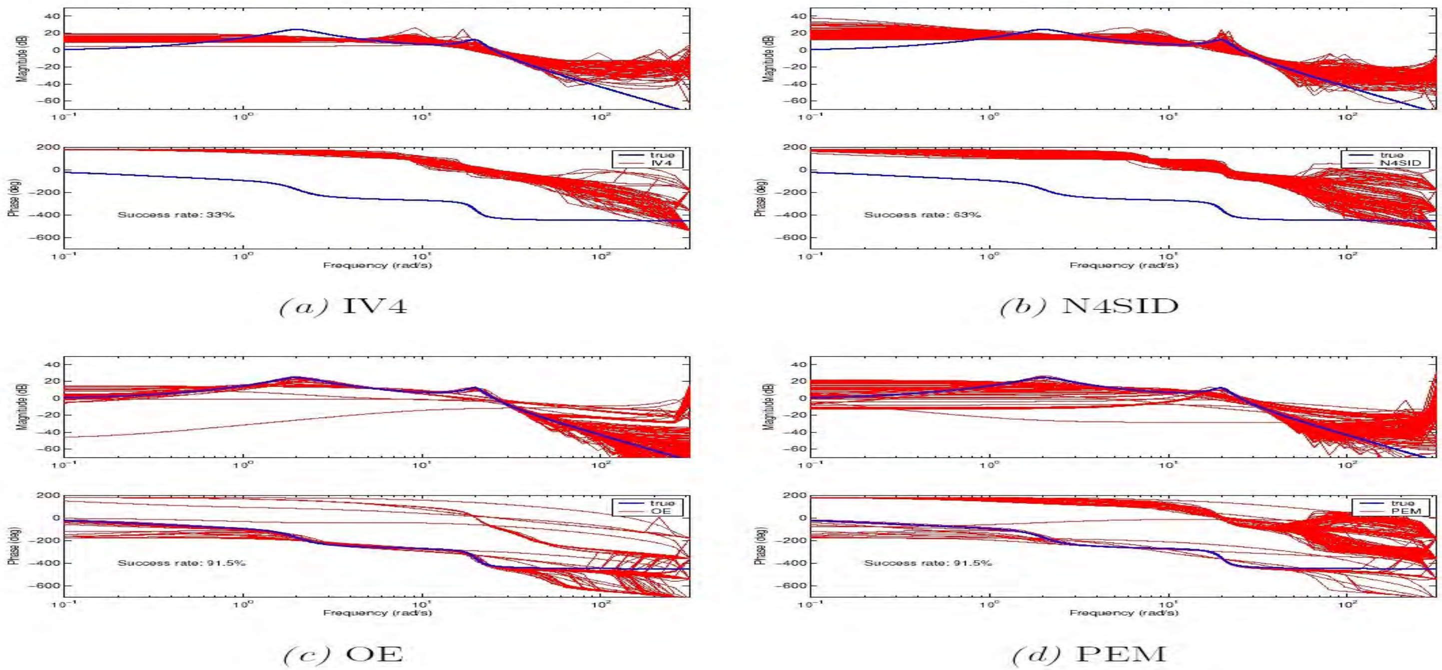
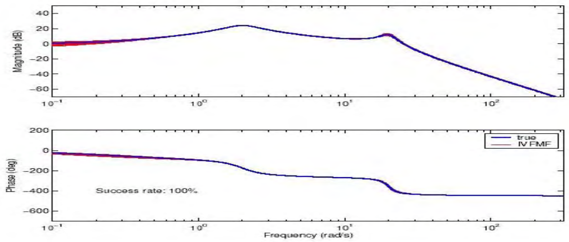
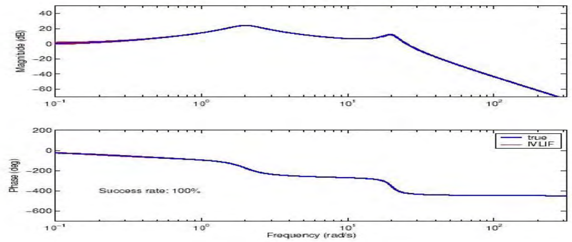


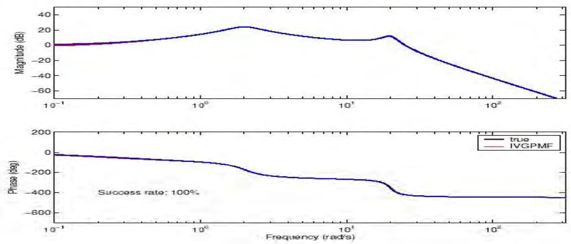
Fig. 7. Bode plots of DT models resulting from DT methods (Trial9)



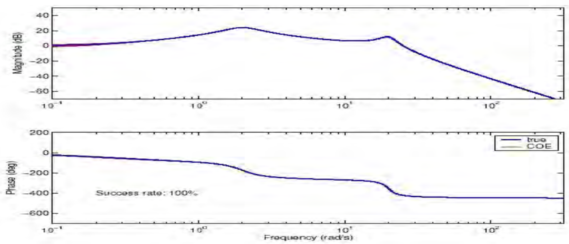
(a) IVFMF



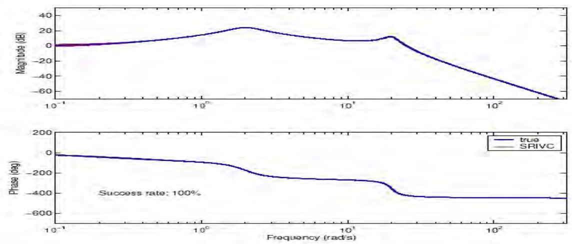
(b) IVLIF



(c) IVGPMF

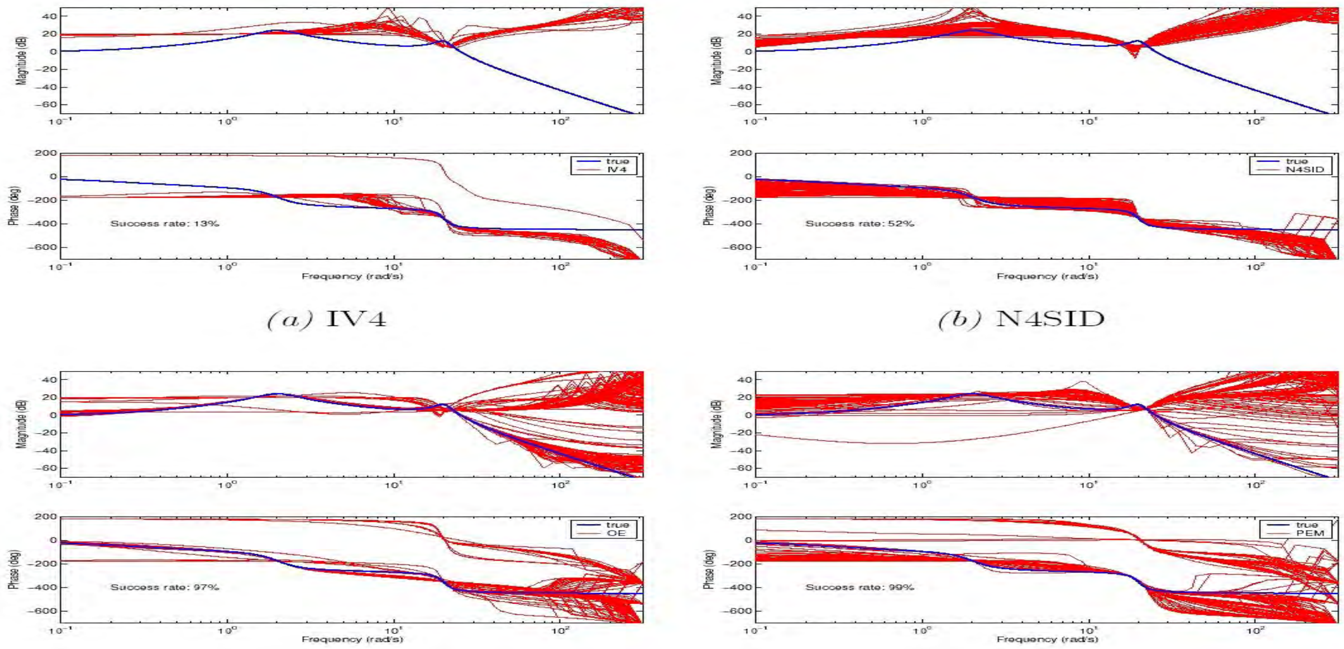


(d) COE



(e) SRIVC

Fig. 8. Bode plots of CT models resulting from direct methods (Trial2)



(a) IV4 *(b)* N4SID *(c)* OE *(d)* PEM

Fig. 9. Bode plots of DT models resulting from DT methods (Trial2)

Paper: ECC, 4th Sept 2003

- INITIALISATION ASPECTS FOR SUBSPACE AND OUTPUT-ERROR IDENTIFICATION METHODS
- - Lennart Ljung

•This paper is inspired by a recent contribution by Rao and Garnier about identification of continuous time models.

•They show examples where methods that directly estimate continuous time models, based on smoothed differentiated input-output data outperform methods that are based on discrete time model estimation.

•The reasons for that situation are investigated in this contribution. It turns out that the key problem is that ARX-type models are very biased for the example in that study, which leads to problems for initializations for output error models both based on ARX/IV techniques and on subspace (CVA estimation techniques).

•The remedy is to decrease the ARX-bias via low pass data filtering, which in turn also explains why the direct continuous-time estimation techniques (with inherent data smoothing) do not suffer from this problem.

A recent paper,

[5](G.P. Rao and H. Garnier. Numerical illustration of the rel- evance of direct continuous-time model identification. In *Proc. 15th IFAC Triennal World Congress*, Barcelona, July 2002.)

shows comparisons between two ways of estimating continuous time models:

- **1. Directly fitting smoothed derivative approximations of in- put and outputs to continuous-time models, e.g. [1], [9]**
- **2. Estimating discrete time models from the data, which are then transformed to continuous time.**

The results show, for the chosen example, that approach (1) is much better than approach (2).

- **This is intriguing, since theoretically the route via discrete time models cannot be inferior to direct fitting.**
- **In this paper we confirm that the selected system in [5] indeed gives severe problems for the basic discrete time identification techniques, including both prediction-error, output error and subspace (CVA/MOESP/N4SID) techniques.**

- **It also turns out that the remedy is to move the focus in the model fit to lower frequencies by proper pre-filtering. Since pre-filtering is inherent in the direct continuous-time techniques this also explains why such initialization problems do not occur for those (CT) techniques**

- **The fact the subspace/CVA estimate is so bad for this particular system should deserve an analysis of its own, since CVA is known to be very reliable in general. The basic reason in this particular case is probably that only 5 sinusoids are exciting the system, so the higher order ARX-models employed by CVA/subspace are not reliable**

- **The true continuous system has only one zero. If we know that for a fact, this obviously has great importance for the model accuracy at high frequencies.**
- **However, this is a difficult constraint to handle in the sampled models. To use it, we could fit directly a continuous time model.**

- The system used by [5] deserves special attention by people who develop discrete-time identification methods.
- Techniques such as CVA/subspace methods and prediction error methods may give quite bad results if not proper data prefiltering is applied.
- We have found that ARX-models are very bias-sensitive to the system (especially with sinusoidal inputs). ----- the bias is substantial, despite the good signal to noise ratio. Even though the bad conditioning of the regression matrix is part of the reason for the bias, it is not a question of numerical errors.
- This means that typical initialization routines based on ARX models will have problems. One should discuss various remedies, in addition to pre-filtering, for this initialization problem.

- This talk is based on the paper
- ***Identification of Continuous-Time Systems, G.P. Rao and H.Unbehauen,*** IET Control Theory & Applications journal (formerly known as IEE Proceedings Control theory & Applications) Vol.153, Issue 2, (March 2006)

Recent letter from the Managing Editor

- (This paper) was among the top 20 most downloaded IET Control Theory & Applications papers by the users of the IEEE Xplore database in 2008.
- Your paper is ranked No. 14 from the hundreds of papers that the journal has published since its launch in 1980, receiving 197 full text downloads last year.

- On behalf of the journal’s Editor-in-chief, Prof. Brett Ninness, I would like to congratulate you on the publication of a paper that is clearly of significant interest to the community. I am pleased that IET Control theory & Applications journal has been able to position your paper in a way that has made it visible to the community, and very much hope that you will continue to contribute to the journal in the future.
- Best regards

Lee Baldwin

Managing editor, IET Control Theory& Applications

Conclusions

- **This lecture has been limited to linear time invariant systems although there are significant results in time varying and nonlinear systems included in the paper.**
- **The direct approach to CT model identification is found to be more dependable. This is not surprising to me because this is a direct route.**
- **I will not be surprised likewise if a ‘direct DT approach’ performs better in the cases where the model is native to the DT domain. But such situations need to be seen.**
- **I hope that more methods will be added in the future to CONTSID toolbox to give a better choice.**

Future

- Identification will be a perennial activity
- Tools need to be researched for enhancement and wider applicability.
- Distributed parameter systems, nonlinear systems need to be considered for inclusion.

Identification=effort to gain knowledge

- New results must therefore be welcome setting aside commercial and group professional interests.
- I hope that future will be based on fairness of assessment and free exchange of ideas for the benefit of all in the scientific community.

Acknowledgements

- It was a challenge to take up research in the 1970s in CT identification in the Digital Age.
- My students and colleagues had shown great faith and worked with me in this field.
- They all contributed greatly to the present state of the field.

Thank you all for your interest and patience

May God Bless You All